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EXPONENTIAL STABILITY OF THE VON KÁRMÁN SYSTEM WITH INTERNAL DAMPING

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Abstract

This work deals with a von Kármán system with internal damping. For the solution's existence, we use nonlinear semigroup theory tools. We construct an evolution system by nonlinear Lipschitz perturbation of a semigroup of contractions. We apply the energy method for the asymptotic behavior, which uses suitable multipliers to construct a Lyapunov functional that leads to exponential decay.

1 Introduction

In this paper, we study the existence of solution and asymptotic behavior for the initial boundary value problem of the von Kármán beam system of the type

$$\begin{cases} \rho A w_{tt} - EA\left[\left(u_x + \dfrac{1}{2}w_x^2\right)w_x\right]_x + EIw_{xxxx} = 0 \quad \text{in} \quad (0, L) \times (0, T), \\ \rho A u_{tt} - EA\left[u_x + \dfrac{1}{2}w_x^2\right]_x = 0 \quad \text{in} \quad (0, L) \times (0, T). \end{cases} \tag{1.1}$$

where $w(x, t)$ is the transverse displacement of a generic point, $u(x, t)$ the longitudinal displacement, $(0, L)$ is the segment occupied by the beam, and T is a given positive time. The physical parameters represent the properties of the material being E the Young's modulus, A the cross-sectional area of the beam, L the beam length, ρA the weight per unit length and EI the beam stiffness or flexural rigidity. The model (1.1) was proposed by J. E. Lagnese and J. L. Lions, see [3, 4].

Here we are interested in studying the existence of solution and asymptotic behavior, considering frictional damping, which is a natural problem, given by

$$\begin{cases} w_{tt} - b_1\left[\left(u_x + \dfrac{1}{2}w_x^2\right)w_x\right]_x + b_2 w_{xxxx} + a_1 w_t = 0 \quad \text{in} \quad (0, L) \times (0, T), \\ u_{tt} - b_1\left[u_x + \dfrac{1}{2}w_x^2\right]_x + a_2 u_t = 0 \quad \text{in} \quad (0, L) \times (0, T). \end{cases} \tag{1.2}$$

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We consider the initial data and boundary conditions, respectively

$$\begin{cases} w(x,\,0)=w_0(x),\ w_t(x,\,0)=w_1(x), \\ u(x,\,0)=u_0(x),\ u_t(x,\,0)=u_1(x), \end{cases} \tag{1.3}$$

$$\begin{cases} u(0,\,t)=u(L,\,t)=0, \\ w(0,\,t)=w(L,\,t)=0, \\ w_x(0,\,t)=w_x(L,\,t)=0. \end{cases} \tag{1.4}$$

Now, we introduce the Hilbert space

$$\mathcal{H}=H_0^2(0,\,L)\times L^2(0,\,L)\times H_0^1(0,\,L)\times L^2(0,\,L),$$

equipped with the inner product given by

$$\langle U,\,\widetilde{U}\rangle_{\mathcal{H}}=b_2\int_0^L w_{xx}\overline{\widetilde{w}}_{xx}dx+\int_0^L \varphi\overline{\widetilde{\varphi}}dx+b_1\int_0^L u_x\overline{\widetilde{u}}_x dx+\int_0^L \psi\overline{\widetilde{\psi}}dx, \tag{1.5}$$

where $U=(w,\,\varphi,\,u,\,\psi)^T$, $\widetilde{U}=(\widetilde{w},\,\widetilde{\varphi},\,\widetilde{u},\,\widetilde{\psi})^T$, we introduce the functions $\varphi=w_t$ and $\psi=u_t$. We now wish to transform the initial boundary value problem (1.2)-(1.4) to an abstract problem in the Hilbert space $\mathcal{H}$. Rewrite the system (1.2)-(1.4) as the following initial value problem

$$\begin{cases} U_t=\mathcal{A}U+\mathcal{F}(U), \\ U(0)=(w_0,\,\varphi_0,\,u_0,\,\psi_0)^T, \quad ,\forall\, t>0, \end{cases} \tag{1.6}$$

The domain of operator $\mathcal{A}:\mathcal{D}(\mathcal{A})\subset\mathcal{H}\to\mathcal{H}$ is given by

$$D(\mathcal{A})=H^4(0,\,L)\cap H_0^2(0,\,L)\times H_0^2(0,\,L)\times H^2(0,\,L)\cap H_0^1(0,\,L)\times H_0^1(0,\,L).$$

The main idea is to consider the nonlinear evolution system (1.6) as a locally Lipschitz perturbation $\mathcal{F}$ of a linear contraction semigroup $S(t)=e^{\mathcal{A}t}$ on $\mathcal{H}$. Since the nonlinear term $\mathcal{F}$ is locally Lipschitz, then abstract results (see [2], Chap. 6 and [5],Theorem 7.1) on the generation of nonlinear semigroups apply in order to conclude the existence of a nonlinear semigroup on $\mathcal{H}$. Nonlinear semigroup theory also implies that for initial data taken from the domain of the generator, the corresponding solutions are continuous in time with the values in $\overline{D(\mathcal{A})}$. For an outline of the proof, see [[6], Appendix]. Thus strong solutions possesses the property $\in C([0,\,T),\mathcal{H})$.

To get $S(t)=e^{\mathcal{A}t}$ on $\mathcal{H}$, we will use the well known the Lumer-Phillips theorem (see [2]) and $\mathcal{F}$ is locally Lipschitz we adapt the idea as in [1], Lemma 3.

Our solution existence result is given by

Theorem 1.1. *If $U_0\in\mathcal{H}$, then problem* (1.6) *has a unique mild solution $U(t)=e^{\mathcal{A}t}U_0+\int_0^t e^{\mathcal{A}(t-s)}\mathcal{F}(U(s))ds$, $U\in C([0,\,\infty):\mathcal{H})$, with $U(0)=U_0$. Moreover, if $U_0\in D(\mathcal{A})$ the mild solution is a strong solution globally defined.*

2 Asymptotic behaviour (Main result)

Theorem 2.1. *Let $(w,\,u)$ be a solution of* (1.2) *where the initial data are given in $D(\mathcal{A})$. Then, the energy $\mathcal{E}(t)$ satisfies* $\quad \mathcal{E}(t)\le C\mathcal{E}(0)e^{-\alpha t},\quad \alpha, C>0,\quad$ for all $t>0$.

References

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